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CASE STUDY FOR VINAYAK RESTAURANT QUEUING MODEL

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ABSTRACT

Waiting time and service system is a part of our daily life in various service areas such as: restaurant, ATM, Clinic, Toll tax, Ticket window in cinema, Bus stand and Railway station. Here, our focus is on Restaurant Management System. Every restaurant loses their customers just because of a long queue and maximum waiting time. Restaurant provides more waiting chairs only. We need to be improving the service time which shows improvement in queuing situation. Here, we apply queuing model. This paper aims to shows the multi-channel queuing model M/M/S. We use M/M/S queuing model since this restaurant has 10 service stations. We have obtained one month daily customers data from a restaurant named "Vinayak restaurant" in Laxmangarh, Sikar city. Using little's theorem and the multi-channel waiting line model M/M/S. we have determined the arrival rate λ service rate μ. The utilization rate ρ and the average waiting time in the queue before getting service. At Vinayak restaurant, the arrival rate λ is 1.40 customers per minute (cpm). Customer waiting time is 10 minute and the utilization rate is 0.49. We have discussed the benefits of applying queuing model to a busy restaurant.

KEYWORDS: *Queue, Little's Theorem, Restaurant M/M/S Queuing Model, Waiting Time, Restaurant Data, (A, B, C, D………. X, Y, Zmanner)*

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INTRODUCTION

Queuing theory is the mathematical study of the congestion and delays of waiting in line. A waiting line system or queuing system is defined when a person or object spends time in waiting line to complete a transaction or activity. A waiting time can be measured by its two extreme point of service: a low level of service and a high level of service. Queue or waiting lines are familiar event to all, waiting line are not only the lines of human beings but also the aeroplane seeking to land at busy airport, ships to unloaded machine parts to be assemblies, cars waiting in traffic, etc. Queuing theory is concerned with the statistical description of the behaviour of queues with finding.

Queuing theory assesses two keys aspects – customer arrival at the facility and service. This theory is used to model and predict waiting time and the number of customer arrival. This paper uses queuing theory to study the waiting line in Vinayak restaurant at Laxmangarh in Sikar. This is the busiest restaurant in Sikar. It provides 60 tables for 52 customers. There are 10 waiters working at one time. During weekdays (Monday to Thursday) there are over 200 customers who comes to the restaurant and during weekends (Friday to Sunday) over 490 customers arrives between 6:00pm to 10:30pm for enjoyment and dinner.

LITERATURE REVIEW

Literature review of queuing research that has been implemented or the model can be used for machine repair. Although there is nothing new about the queue, the study of queues is modern phenomenon where it was dating early in twentieth century. It was initially proposed by the Danish telephone engineer A.K. Erlang in 1903. He raised the problem over telephone traffic congestion. Leading investigation Danish mathematician A.K. Erlang has written the theory of probability and telephone conversation in 1909. The area of telephone traffic was ahead developed by Molins in 1927 and Theornton D-Fry in 1928. It started only after World War II. The queue was extended to other common problems. His works inspired engineers, mathematicians to deal with queuing problem using probabilistic methods. Queuing theory originated as a very practical subject. Theoretical analysis of the queue system increased significantly with the advent of Operations Research in the late 1940s and early 1950s. The first text book on the subject queues, inventory and maintenance was written by Morse in 1958. T.L. Satty wrote her famous book "Element of Queuing Theory with Applications in 1961 and Kynrock completed their queuing system in 1975. Queuing theory became a field of applied probability and many of its results have been used in Operations Research, computer science, telecommunication, traffic engineering and reliability theory. It should be emphasized that there is a living branch of science where specialists publish lots of paper and books.

Basic Characteristics

A queue system fulfilled by the following basic characteristics is specified: input process, queue discipline, service mechanism, capacity of the system, service channel.

At a service center, there are one or more service channels or service stations.

- If the number of server is infinite, then all the customers are served instant on arrival and there will be no queue. Every customer takes service without any wait.
- If the number of server is finite, then the customers are served according to a specific order.
- If the service stations are empty, then the arriving customers will be served immediately. If service is not available then the arriving customer will not wait anymore. Customer leaves the system after taking service.

Characteristics of a Queuing System

Queuing model have two main elements- customer and server. Generally, customer is called units. Customer may be a person, machine, vehicles, and etc. i.e. the units will be the same as a system will be. A server is the system that provides service to customer; this may be single or multi channel. Customer originates from a single source. If the customer are equal and less than to server then the server provides service to the customer immediately. If the customer is more than the server, then the customer waits to receive service. If there is more than one waiting customer, then the waiting customer makes a queue. When server completes service, it automatically pulls a waiting customer from the queue, if any. If the queue is empty, then the server becomes inactive until a new customer arrives. A queuing model is completely specified by 6 characteristics:-

- Arrival time distribution: - it represents the incoming pattern of the customer in the system. In a given time, the number of arrivals is estimated by using a discrete probability distribution. The arrival time follow Poisson distribution.
- Service time distribution: - it represents the pattern in which the number of customer leaves the system. A description of the resources needed for service to begin. It is independent of the inter-arrival time. It follow exponentially probability distribution.
- Service channel: - the waiting line system have two types of service channel-

Single Service Channel: in this channel, the system has only one server. In the system number of service channel which may be arranged in parallel or in series or a complex combination of both.

Multi Service Channel : in this channel, the system has s server. The system has a number of parallel channels each with a server.

- Queue discipline: - there are several possibilities in terms of the sequence of customer to be served such as- FCFO (first come first out); these disciplines are common e.g. railway stations, doctor's clinic. LIFO (last in first out) i.e. the last one to come will be the first to be served like in a big warehouse and SIROC (service in random order) based on priority.
- Queue lengths: - queue in a system can be modelled as an infinite or finite queue length. A limited source range customer arrival for service.
- System capacity: - the maximum number of customer in a system can either be finite or infinite. It also includes customers waiting in queue. Only limited numbers of customer are allowed in limited facilities, system and new arrivals. It is not allowed to join the system until it falls below the specified number.

Kendall's Notation

Queuing model can be expanding in the following symbolic form- (a / b / c): (d / e / f)

Where

- a = arrival time
- b = service time distribution
- c = number of channels (1, 2, 3, 4 …)
- d = capacity of system
- e = queue discipline
- f = size of calling source

The above notation is known as Kendall's notation. First three characteristic in the above notation i.e. (a/b/c) were introduced by D. Kendall in 1953 and the remaining notation i.e. (d/e/f) added by A. Lee in 1983.

This notation is not suitable for describing complex models i.e. queue in series or network queue.

Little's Theorem

Little's theorem is given by John Little. This theorem describes the relationship between arrival and service rate. The average number of customer in the system is equal to arrival rate λ multiplied by the average waiting time in the queue which can be written as –

$$L_s = \lambda W_s \tag{1}$$

Here – λ = mean arrival rate for customers coming to the system.

W_s = expected waiting time in the system.

The result applies to any system and particularly. It applies to system within system.

There are three fundamental relationships which can be derived from little's theorem –

- If λ or W_s increase then L_s is also increase.
- If L_s increase or W_s decrease then λ increase.
- If L_s increase or λ decrease then W_s increase.

Vinayak Restaurant Model (M / M / S: FCFS/∞)

We observe one month daily customer's data with the help of restaurant manager. We know all about the restaurant capacity and the number of waiters and waitress by the help of restaurant manager. Based on the above information, we have decided that the queuing is (M/M/S: FCFS/∞).

In this model, the customer arrive in a Poisson distribution with mean arrival rate λ. S is fixed which states the number of service station. Service stations are arranged in parallel, if system is parallel, then a customer can go to any of the free counters for his service. The service time of each counter is identical and follows the same exponential distribution. The mean service rate per busy service is μ. Restaurant has 10 server i.e. S=10.

Here, S=10 and $S \geq n$.

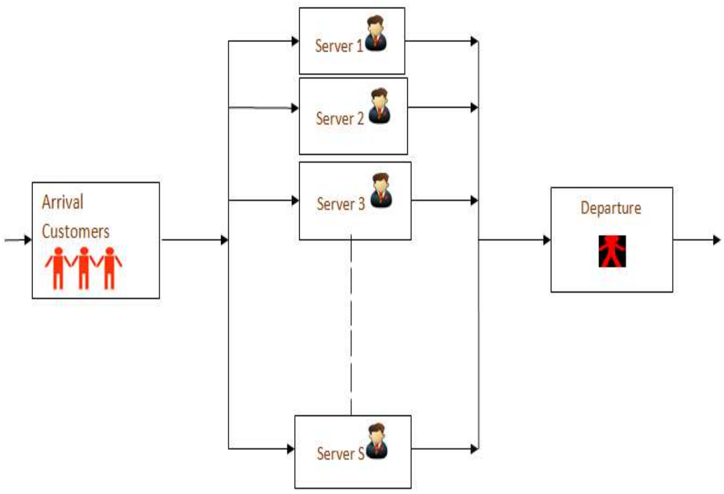

Figure1: Representation of M / M / S Queuing System.

Assumption

- Customer comes from an infinite population.
- They follow the Poisson distribution.
- A service discipline is FCFS i.e. first come first service.
- Service time follows the exponential distribution.

When there are n units in the system, we have two situations for service rate μ-

- If S ≥ n, the entire customer may be served simultaneously in this situation. System will be in no queue and
- (S-n) number of server remains in the system.
- $\mu_n = n\mu$ for n = 0,1,2,3………..S.
- If n ≥ S, in this situation, all server are busy and system will be in queue and maximum number of customer which are waiting in queue i.e. (n-S).

  $\mu_n = S\mu$

Here, this paper aims that all customers should get service and the waiting time of customer is minimum in the queue. So, we take $\mu_n = n\mu$ (S ≥ n).

The utilization factor ρ <1 or $\frac{\lambda}{n\mu}$<1 i.e. the average service take is faster than the average arrival rate.

For analysing the Vinayak restaurant M/M/S queuing model the following variables. There we will be investigation variables.

λ = λ, the mean customer's arrival rate.

$\mu_n = n\mu$,(for n = 0,1,2,3,………,S) if S ≥ n, the mean service rate. (2)

$\rho = \frac{\lambda}{n\mu}$,, the utilization factor. (3)

Probability of zero customer in the restaurant if S ≥ n and ρ < 1-

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{1}{(1-\rho)}\right]^{-1} \tag{4}$$

If ρ >1 then probability of zero customer is-

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{1}{(\rho-1)}\right]^{-1} \tag{5}$$

The probability of having n customer in the restaurant is S ≥ n –

$$P_n = \left[\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] P_0 \tag{6}$$

L_q: the average number of customer in the queue-

$$L_q = \left[\frac{\rho(s\rho)^s}{s!(1-\rho)^2}\right] P_0 \tag{7}$$

L_s : average number of customer in the restaurant –

$$L_s = Lq + \frac{\lambda}{\mu} \tag{8}$$

$$L_s = \left[\frac{\rho(s\rho)^s}{s!(1-\rho)^2}\right]P_0 + \frac{\lambda}{\mu}$$

W_q : the average waiting time in the queue –

$$W_q = \frac{Lq}{\lambda} \tag{9}$$

$$W_q = \left[\frac{(s\rho)^s}{n\mu s!(1-\rho)^2}\right]P_0$$

W_s : the average waiting time spent in the restaurant –

$$W_s = \frac{Ls}{\lambda} \tag{10}$$

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = \left[\frac{(s\rho)^s}{n\mu s!(1-\rho)^2}\right]P_0 + \frac{1}{\mu}$$

OBSERVATION AND DISCUSSIONS

The one month daily customer's data were shared by the restaurant manager as shown in Table 1

Table 1

Week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1^{st} week	250	208	290	270	500	550	595
2^{nd} week	210	280	227	255	539	545	530
3^{rd} week	230	225	282	229	540	582	590
4^{th} week	300	231	249	249	590	600	571

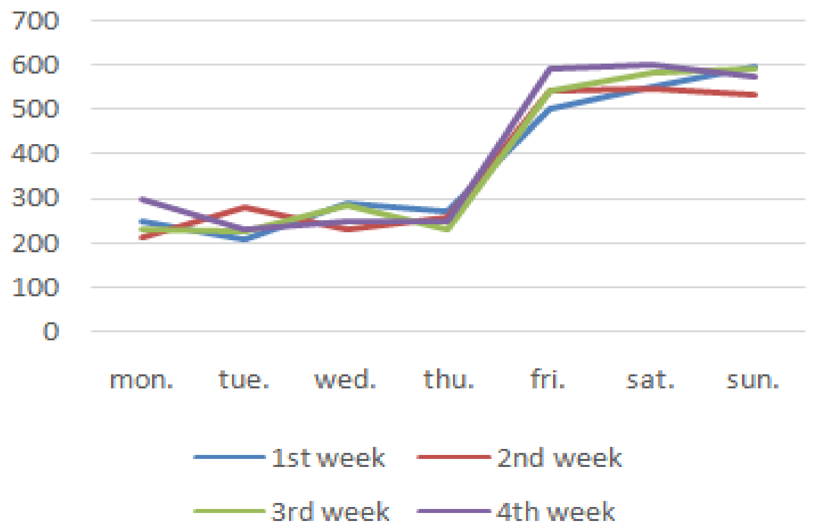

Figure 2: One Month Daily Customer Counts.

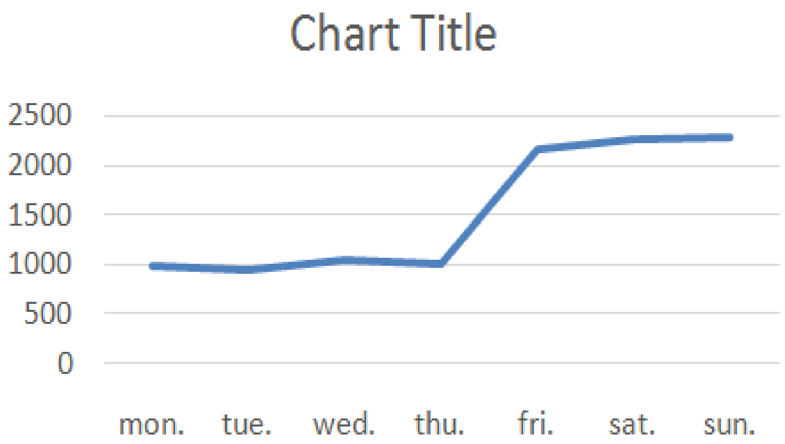

Figure 3: One Month Total Customer Counts.

Calculation

On average, there are 380 customer comes to the restaurant in 4.5 hours during dinner time. So, the arrival rate is-

$$\lambda = \frac{380}{270} = 1.40 \text{ customer per minute (cpm)}$$

From the observation and discussion, we have found out that on average customer spends 30 minute (W_s) in the restaurant. Queue length is around 15 customers (L_q). On average, waiting time is around 10 minute.

Now, using eq^n (9) –

$$W_q = \frac{15}{1.40} = 10.71 \text{ minute}$$

It is noted that if we compare the actual waiting time and theoretical waiting time then the difference between both times does not very much.

The average number of customer in the Vinayak restaurant. Using little's theorem using (1) -

$$L_s = \lambda W_s = 1.40*30 = 42 \text{ customer}$$

We can derive the service rate by using eq^n (8) –

$$L_s = Lq + \frac{\lambda}{\mu}$$

$$\frac{1}{\mu} = \frac{1}{\lambda}[Ls - Lq] \quad = \frac{1}{1.40}[42 - 15]$$

$$.\frac{1}{\mu} = 19.28$$

$$.\mu = 0.051$$

Here, we use $\mu = n\mu$ for n = 0, 1, 2, 3… S.

$$\text{So, } n\mu = \sum_{n=0}^{S} n * \mu = \sum_{n=0}^{10} n * 0.051 = 2.805$$

Hence, using eq^n (3) –

$\rho = \frac{\lambda}{n\mu} = \frac{1.40}{2.805} = 0.4991$

With high Utilization factor 0.49 during dinner period. The probability of having zero customer in the restaurant here, $\rho < 1$ so we use eq^n (4) -

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \frac{1}{(1-\rho)}\right]^{-1}$$

$P_0 = 2.81$

i.e. probability of having zero customer in the restaurant is 2.81.

Evaluation

- The utilization factor i.e. $\rho = \frac{\lambda}{n\mu}$ where λ is a mean number of customer's arrival rate and μ is a mean service rate and n = 0, 1, 2 ….S.
- If the mean number of customer will increase, then the utilization factor will also increase. So, we can say that the utilization factor is directly proportional to the mean number of customer i.e. $\rho \propto \lambda$.
- If the service rate increases then the utilization factor decreases. We can say that the utilization factor is inversely proportional to the service rate i.e. $\rho \propto \frac{1}{\mu}$ ..
- In this restaurant, the utilization factor is 0.49 which is high.
- In case where the customer's waiting time is less than 10 minute, then the number of customer that are served per minute in the system will increase.

Benefits

As Vinayak restaurant is the busiest restaurant in Sikar city, this paper (research) can help this restaurant to ascertain their QOS (quality of service) to increase. If there are many customers in the queue, A model can be created to make this improvement, as now the restaurant can estimate how many customers will be waiting in the queue, leading to a large number of customer projected to arrive at the system with a view to receiving service. The formulas provide a network for creating restaurant queues that are simpler than construction.

CONCLUSIONS

This research paper has discussed the application of multi-channel M/M/S queuing model in Vinayak restaurant in Sikar. We applied the M/M/S model. We have obtained the customer's arrival rate λ is 1.40 cpm and here, we take service rate i.e. $\mu_n = n\mu$, (for n = 0, 1, 2, 3… S).If $S \geq n$ [using eq^n (2)] so, $n\mu = \sum_{n=0}^{S} n * \mu$ is 2.805. And, the probability of having zero customers in the restaurant is 2.81. As our future work, we will develop a simulation model for the restaurant. By developing a simulation model, we will be able to confirm the results of the analytical model which we have developed in this paper. Queuing analysis is one of the most practical and effective tools for understanding and aiding decision making in managing critical resources.

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